STAT139 Final- Group 6
Does money buy happiness? What affects world happiness?

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1 Introduction and Motivation

Everyone knows the adage "money can't buy happiness." It is clear that being wealthy does not guarantee happiness, but there are many data showing the positive relationships between money and happiness. Study from Daniel Kahneman and Angus Deaton indicates that emotional wellbeing rises with income. However, it rises logarithmically. As an individual's income increases, their wellbeing increases at a slower and slower rate[5]. Research from Christopher J. Boyce and other scholars shows that income rank, instead of income, affects life satisfaction [3]. Other scholars found that how to use money is more important and spending money on others may promote happiness [4]. There are many researches about happiness on individual level, but fewer research about the happiness on the country level. At the country level, GDP, Gross domestic product, is an importance economic metric of the country's wealth level. Over the past decades, the average world GDP increase tremendously, though varying across different countries. This naturally leads us to think about whether the increase of world wealth level increases world happiness, and whether there are other factors that affect the world happiness. The World Happiness Report [2] evaluates the state of global happiness, and gives each country a happiness score per year. Therefore, in our analysis below, we would use the happiness score from World Happiness Report as our response and explore factors including GDP that are possibly related to happiness at the country level. Our main interested questions are as below:

(1) How does happiness score vary by GDP? Is higher GDP positively correlated with higher happiness?

(2) How is the relationship between GDP and happiness affected after adding other factors into the models? Are there other factors that have influence on country-level happiness?

(3) How does the relationship between factors and happiness score further change if country is added as a random effect into our model?

2 Data

2.1 Data Source and Description

Our data comes from the World Happiness Report mentioned above. The report was first published in 2012 with data collected in 2011. Since then, the report has been published annually, except for 2014. As time passes by, the World Happiness Report continues to gain global attention. The 2017 Report is even featured on an event held by the United Nation to celebrate the International Day of Happiness on March 20th. The happiness score in the Report has been growingly used by government agencies and non-profit organizations as a measure of their policy-making decisions.

The data is downloaded directly from the Data Panel section of the World Happiness Report website2. Each observation in the data represents a country and the number of countries differs across years. Though the year of the data ranges from 2005 to 2020, we chose to use data from 2006 to 2020 due to serious missing data problems in the other years. Our response variable is happiness score (or life ladder). Our exploratory variables measuring the state of happiness include log GDP per capita, social support, healthy life expectancy at birth, freedom to make life choices, generosity, perceptions of corruption, positive affect, negative affect, year, and country name. A detailed description of our variables can be found in Appendix A. In total, we have 1922 observations and 11 variables after dropping all observations with missing data in any variables.

2.2 Data Pre-processing

In the dataset, there are there are 9 numerical variables and 2 categorical variables ('year', 'Country.name'). We first transform the types of 2 categorical variables into 'factor'. As for the missing values in the data set, there are 8 columns with missing values and the column with the most missing values is 'Perceptions.of.corruption'[Figure 1 left], which contains 108 missing values (5.62% of total data). Among the 21 missing patterns [See Figure 1 right], there is only two patterns that have more than 4 missing variables. Except this pattern, there are no more than three variables missing. Since the missing value problem is not serious, so we will not apply any imputation but directly drop those samples. After dropping the missing value, there are 1707 samples remaining.

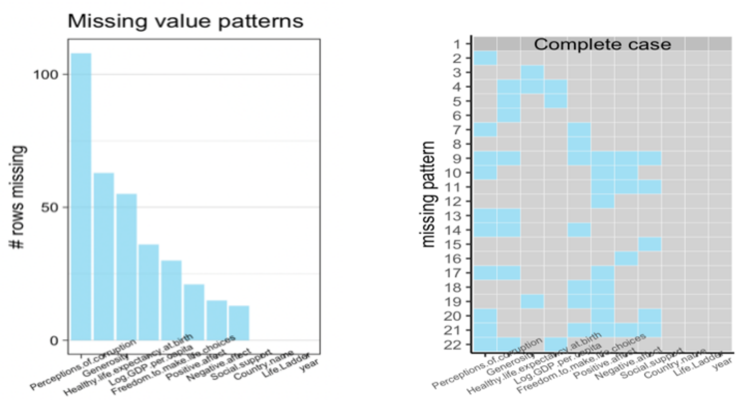

Figure 1: Missing Patterns for Different Variables

2.3 Exploratory Data Analysis

Distributions of Variables

We first look at the histograms of all the variables [Figure 4]. Some histograms, like 'social.support' and 'Perceptions.of.corruption' are kind of left-skewed. However, in order to achieve higher interpretation ability, we don't require histograms of each predictor in perfect bell shape and decide not to do transformation here.

Correlation Heat-Map

We then draw the correlation heat-map among all the variables, including response and predictors [Figure 2].From the correlation plot, we could see that the linear relationship between 'Life.Ladder' and 'Log.GDP.per.capita' is the most obvious, so we will explore the relationship between GDP and country-level happiness score first in our model part. There are also positive linear relationships between 'Life.Ladder' VS 'Social.support' and 'Life.Ladder' VS 'Healthy.life.expectancy.at.birth', which supports the viability of linear models. Besides, there seems to be linear relationships between predictors as well, like 'Log.GDP.per.capita' and 'Healthy.life.expectancy.at.birth', which may cause multicollinearity problem.

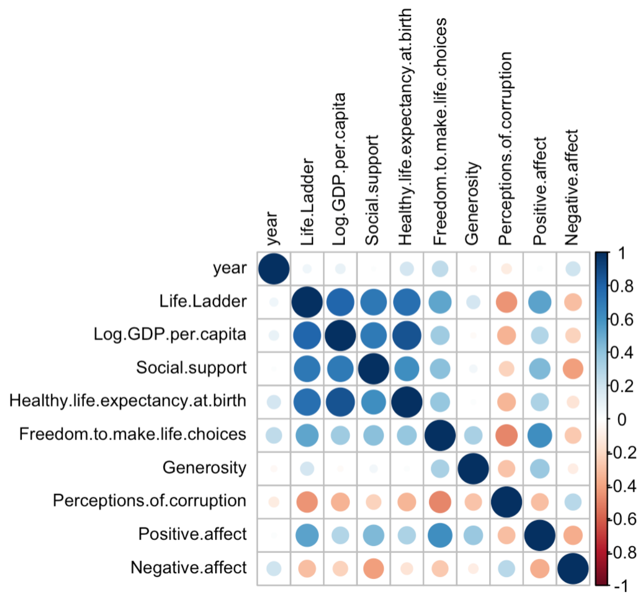

Figure 2: Correlation HeatMap

Happiness Score On Different Countries

From our common sense, there would be lots of country-level factors involving cultural, economic situation and etc. that would greatly affect country-level happiness, so different countries probably have different happiness scores. We calculate each country's average happiness scores over the 15 years and draw the frequency plot [Figure 3]. From the plot, we could see that average happiness scores over years of each country are scattered and have a wide range. So later in our model part, we would also add 'country' as a random effect to consider its impact.

Time Trends of Variables Over Year

Global economic, cultural and political situations change tremendously over the past decades. To see how global patterns change over time, we calculate the average values in these factors per year across all the countries and draw the time-trend plots from year 2006 to year 2021[Figure 5,6]. From these plots, we see that some factors across all the countries fluctuate over years, such as "Social.support", "Generosity" and "Positive emotion measurement". While there are some obvious increasing patterns over years in factors like happiness score, GDP values, and healthy life expectancy, which makes sense and matches our initial research motivation. Factors like the extent of satisfaction with the freedom to choose the lifestyle and negative emotion measurement, drop a little during the first five years, and then increases over the rest years.

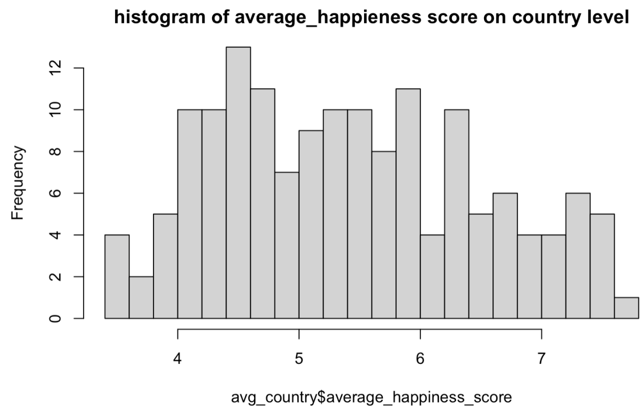

Figure 3: Average Happiness Scores Across Years On Different Countries

3 Methods

In this section, we will first try Ordinary Least Squares Models(OLS) and then Mixed effect models to analyze the relation between happiness score and other economic factors, social factors and etc. We will include the following ten predictors: Country.name(Country), Log.GDP.per.capita (X_1), Social.support(X_2), Healthy.life.expectancy.at.birth(X_3), Freedom.to.make.life.choices(X_4), Generosity(X_5), Perceptions.of.corruption (X_6), Positive.affect(X_7) Negative.affect(X_8), year(X_9). The response variable is Life.Ladder(Y) for all models (see Appendix for detail description).

3.1 Ordinary Least Squares Models

Simple Regression Model (model1)

Ordinary least squares (OLS) is used for estimating the unknown parameters in a linear regression model. By minimizing the sum of the square of the difference between the observed response and the predicted response from linear model[1]. The OLS model should satisfy four assumptions: linearity, normality, constant variance and independence.

We first consider the simplest OLS model with single predictor **log.GDP** and the formula is:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \varepsilon_i \tag{1}$$

where $\varepsilon_i \sim N(0, \sigma^2)$.

This simple linear regression is mainly used for examining our first hypothesis about the relationship between Life Ladder and GDP per capita (log scale).

Full Regression Model (model2)

Next, we would like to know how does the relationship between Life Ladder and GDP per capita changes as more predictors are introduced into the model. So we would build a multiple linear regression to investigate the relationship between Life Ladder and all the other numerical predictors (excluding year and country name).

$$Y_i = \beta_0 + \beta_1 X_{1i} + ... + \beta_8 X_{8i} + \varepsilon_i \tag{2}$$

where $\varepsilon_i \sim N(0, \sigma^2)$.

Full Regression Model with Time Effect (model3)

In the third linear regression model, we want to see how the relationship between Life Ladder and other numerical predictors further change as "year" is added into our model. So we would include "year" in our multiple regression model and further consider time effect based on our previous analysis. Nine predictors in total will be considered in the multiple linear regression model, except for country:

$$Y_i = \beta_0 + \beta_1 X_{1i} + ... + \beta_9 X_{9i} + \varepsilon_i \tag{3}$$

where $\varepsilon_i \sim N(0, \sigma^2)$.

3.2 Mixed-Effect Models

In the previous section, we ignore the country effect in all of our OLS models. However countries differ a lot in many ways, and the relation between happiness score and these factors may also be significantly different. For example, even among developed countries with high GDP, the relation between GDP and happiness score still differs a lot[6]. Besides, from our EDA part, we could also see that happiness scores across different countries are quite different. Therefore, it is appropriate to introduce country as a random effect. We are curious to know how the relationship between Life Ladder and all of our predictors change after country is added as a random intercept effect. Specifically, do some predictors lose their significance after country is added into our model?

In this section, we will build two mixed-effect models. In the first mixed-effect model, we would add country as a random intercept effect into our model. In the second mixed-effect model, we would add the variable "country" as a random intercept effect and would also consider random slope effect of other predictors across all the countries.

Random Intercept Mixed Effect Model - Countries (model4)

We consider "country" as a random intercept model.

$$Y_{ij} = \alpha_j + \beta_1 X_{1ij} + \beta_2 X_{2ij} + ... + \beta_9 X_{9ij} + \varepsilon_{ij} \tag{4}$$

$\varepsilon_{ij} \sim N(0, \sigma_Y^2)$, $\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2)$, $i = 1, ..., n_j$ and $j = 1, ..., J$ where n_j is the number of samples of country j and J is the number of countries.

Random Intercept and Random Slope Mixed Effect Model (model5a&5b)

Besides considering "country" as a random intercept effect, we could consider random slope effect on other predictors. A sample model formula which adds a random slope effect on the first predictor is as below.

$$Y_{ij} = \alpha_j + \beta_{1j} X_{1ij} + \beta_2 X_{2ij} + ... + \beta_9 X_{9ij} + \varepsilon_{ij} \tag{5}$$

$\varepsilon_{ij} \sim N(0, \sigma_Y^2)$, $\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2)$, $\beta_{1j} \sim N(\mu_\beta, \sigma_\beta^2)$, $i = 1, ..., n_j$ and $j = 1, ..., J$ where n_j is the number of samples of country j and J is the number of countries.

3.3 Model Comparison

To compare our OLS models above, we could use ESS F-test. To compare our mixed effect models above, we could use mixed model ANOVA test. A mixed model ANOVA is a combination of a between-unit ANOVA and a within-unit ANOVA. It requires a minimum of two categorical independent variables, sometimes called factors, and at least one of these variables has to vary between-units and at least one of them has to vary within-units.

4 Results

Results are presented in two parts. First, how the relationship between happiness scores and all the other predictors looks like as we add more predictors into our OLS model (Table 1). Second, with country added as a random effect into our model, how the relation between happiness scores and other predictors changes (Table 2, Table 3).

4.1 Ordinary Least Squares Models

In the simple linear regression model with only predictor "GDP", the relationship between happiness scores and GDP is positive and significant(at the $\alpha = 5\%$ significance level). One unit increase in the GDP per capita (log scale) is associated with 0.7774 increase in happiness score [Figure 4].

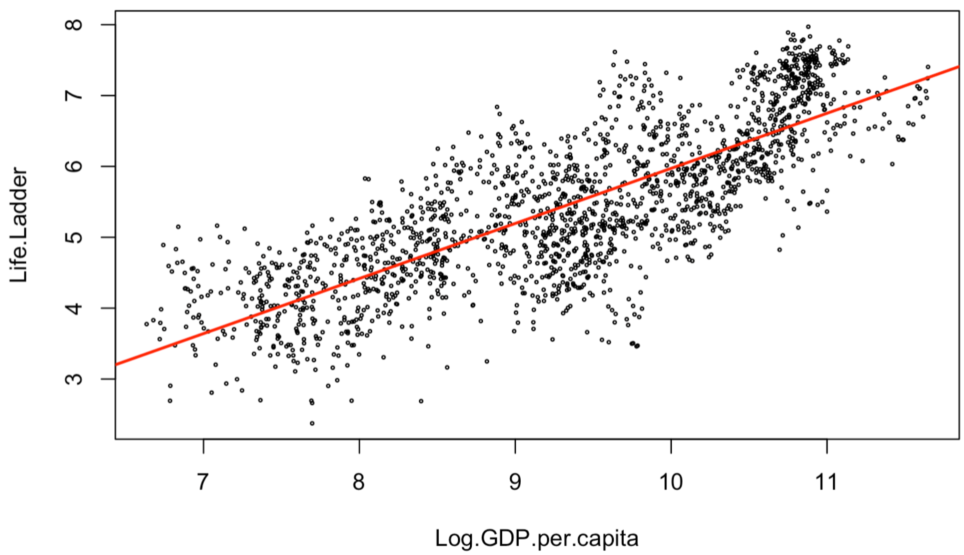

Figure 4: Relationship between log GDP per capita and Happiness Score

As we add other numerical predictors (except for year and country) into our linear regression model, the relationship between happiness score and GDP remains to be positive and significant. However, the coefficient estimate of GDP per capita (log scale) decreases from 0.7774 to 0.3823, which is probably due to the positive correlation between "GDP" and other predictors, like life-expectancy and social support. Moreover, we notice that most of our predictors have significant relationships with country-level happiness scores, and all the significant predictors in our multiple linear regression model have reasonable coefficient signs. The only predictor that is not significant is Negative Affect (a measurement of negative emotions).

Next, we considered time effect and added variable "Year" into our multiple linear regression model. All the significant predictors in our previous model remain significant and their coefficient signs do not change. Negative Affect is still not significant. Coefficients of predictors, "Freedom.to.make.life.choices" and "Negative.effect" change the most after adding variable "year", which is probably because "year" is most closely correlated with these two predictors seen from the correlation heat-map above. Variable "Year" is slightly negatively and significantly associated with happiness scores, which does not match our time trend plot in the EDA part, but its coefficient is very close to 0.

4.2 Mixed-Effect Models

This section is about the results of the mixed-effect models. We tried two types of mixed-effect models with country as random effect: the mixed-effect model with random intercept and the mixed-effect model with both random intercept and random slope of one predictor.

Regarding the model with random intercept(4), some countries have very small intercept such as Sri Lanka(-3.047) but some countries have very large intercepts such as Costa Rica(-0.73) [Figure 5]. The average slopes of log GDP, year, social support, freedom to make life choices, generosity and

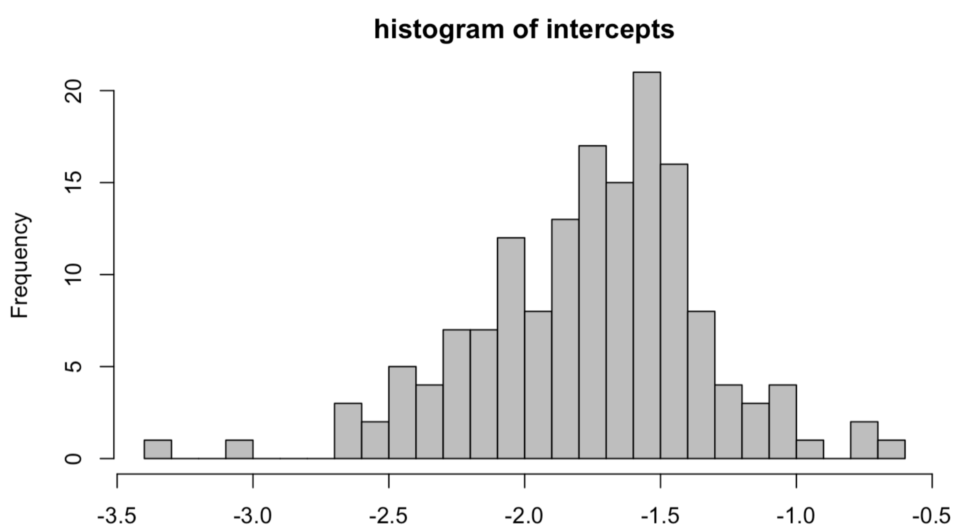

Figure 5: Histogram of intercepts in model 4

positive affect are all positive. The average slopes of Healthy life expectancy at birth, perceptions of corruption and negative affect are all negative. Compared with OLS model with all variables as predictors(3), the sign of Healthy life negative affect and year are reversed as negative, which is more reasonable. However, the sign of Healthy life expectancy at birth is reversed as negative as well, which is unexpected and we will talk about this more in the Discussion section.

Due to this unexpected coefficient, we build second type mixed effect model with both random intercept and random slope. In order to make a comparison, we build two models: one with random slope on log GDP(model5a) and one with random slope on Healthy life expectancy at birth(model5b). Then we do ANOVA test on these two models with model4(the model with only random intercept) separately. The ANOVA test results shows that adding the random slope of log GDP does not significantly increase the explaining power but adding the random slope of Healthy life expectancy at birth can increase the explaining power significantly(see Appendix for detail). From the histogram of log GDP coefficient from model5a[Figure 6], the distribution almost follows a normal distribution. But the histogram of Healthy life expectancy at birth coefficient

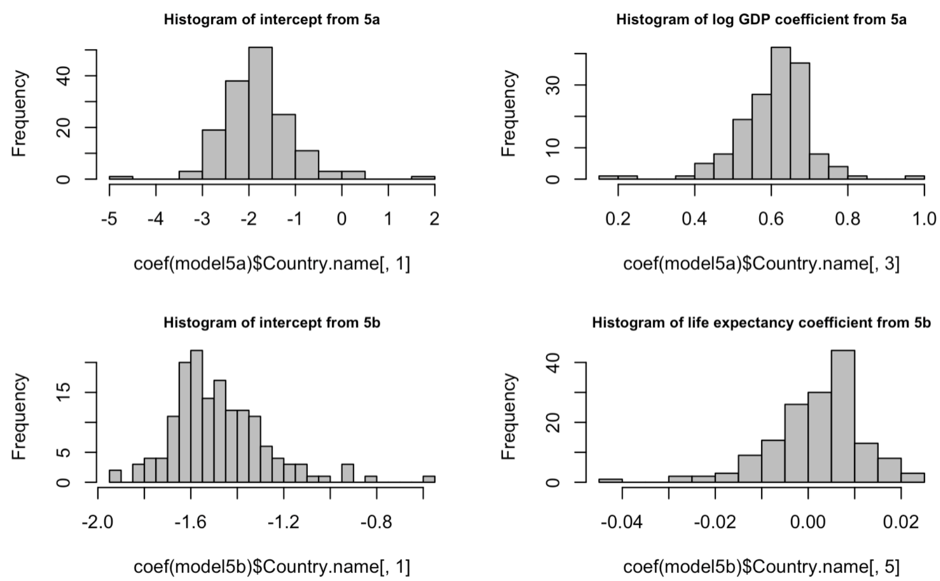

Figure 6: Histogram of intercepts and coefficients in model 5

from model5b is left skewed[Figure 6]. Among all 155 countries, 98 countries hold positive relation between Healthy life expectancy at birth and happiness score and the rest 57 countries hold negative relation between Healthy life expectancy at birth and happiness score. The maximum slope is 0.0238 but the minimum slope is -0.0418. This shows that the huge difference between people's attitude to life expectancy in different countries, which will be talked more in detail in the Discussion section.

5 Discussion and Conclusion

5.1 Discussion of the results

From the model4(the model with random intercept) in Results section, we notice that there is difference between the intercept of each countries. This means the base happiness extend of each country is different. The model contains factors from different aspects: economy, health care, social support, government influence and etc. Even all these factors are the same, people from different countries still show different happiness level. There may be some historical and cultural reasons behind this phenomenon. It is important for people to think what really brings happiness to our life.

From the model 5 (the model with random intercept and random slope) in the Results section, we notice that instead of GDP, life expectancy vary a lot in different countries. With all other factors as the same, people in some countries do not feel happier when the expect length health life become longer. From the result, we find that developed countries usually have positive relation between happiness score and health life expectancy such as Australia, Denmark and Germany. But some underdeveloped countries such as Azerbaijan, Madagascar and Indonesia usually have negative relation between the two variables. This may be highly related to people's life quality, mental stress and the overall social environment.

5.2 Challenges and Limitations

There are several challenges and limitations in this study. First, the number of observations collected in each year is different. Some countries only have two or three observations. Although we tried to overcome this problem by mixed effect model, it would still be better if more data are collected. Second, Since almost all the predictors are significant, we include all the predictors in our models. Thus, it is difficult for to use a 2D graph to visualize the result from the models. Third, there are limit number of predictors in our data set. The study would be more comprehensive and persuasive if more predictors are included.

5.3 Conclusion

The GDP of countries significantly affects country-level happiness. Higher GDP values are positively correlated with higher happiness. This shows us the importance of country's economic situation on people's happiness level. At country level, it seems that money do buy happiness. After adding other factors into the model, the relationship between GDP and happiness is still significant and positive. Many other cultural, and social factors also have significant influence on happiness, such as percentage of people who have social support and the extent of satisfaction with the freedom to choose the lifestyle. The happiness scores at different countries are different, and the impact of life expectancy varies across country.

References

[1] Ordinary least squares, Dec 2021.

[2] Admin. World happiness report 2021: United nations' sustainable development solutions network, Jun 2021.

[3] Gordon D.A. Brown Christopher J. Boyce. Money and happiness: Rank of income, not income, affects life satisfaction - christopher j. boyce, gordon d.a. brown, simon c. moore, 2010.

[4] Elizabeth W. Dunn, Lara B. Aknin, and Michael I. Norton. Spending money on others promotes happiness. *Science*, 319(5870):1687–1688, 2008.

[5] D. Kahneman and A. Deaton. High income improves evaluation of life but not emotional well-being. *Proceedings of the National Academy of Sciences*, 107(38):16489–16493, 2010.

[6] Esteban Ortiz-Ospina and Max Roser. Happiness and life satisfaction, May 2013.

Appendix

Response Variable
Life.Ladder (Happiness score): the national average response to the question of life evaluations. The English wording of the question is "Please imagine a ladder, with steps numbered from 0 at the bottom to 10 at the top. The top of the ladder represents the best possible life for you and the bottom of the ladder represents the worst possible life for you. On which step of the ladder would you say you personally feel you stand at this time?"
Exploratory Variables
Log.GDP.per.capita: Log of GDP per capita **Social.support**: Percentage of people who have social support **Healthy.life.expectancy.at.birth**: Healthy life expectancy at birth **Freedom.to.make.life.choices**: The extent of satisfaction with the freedom to choose the lifestyle **Generosity**: The gap between real average donation and expected donation predicted by GDP **Perceptions.of.corruption**: Trust degree to the government **Positive.affect:** Positive emotion measurement **Negative.affect:** Negative emotion measurement **Year:** from 2006 to 2020

Figure 7: Variable Description

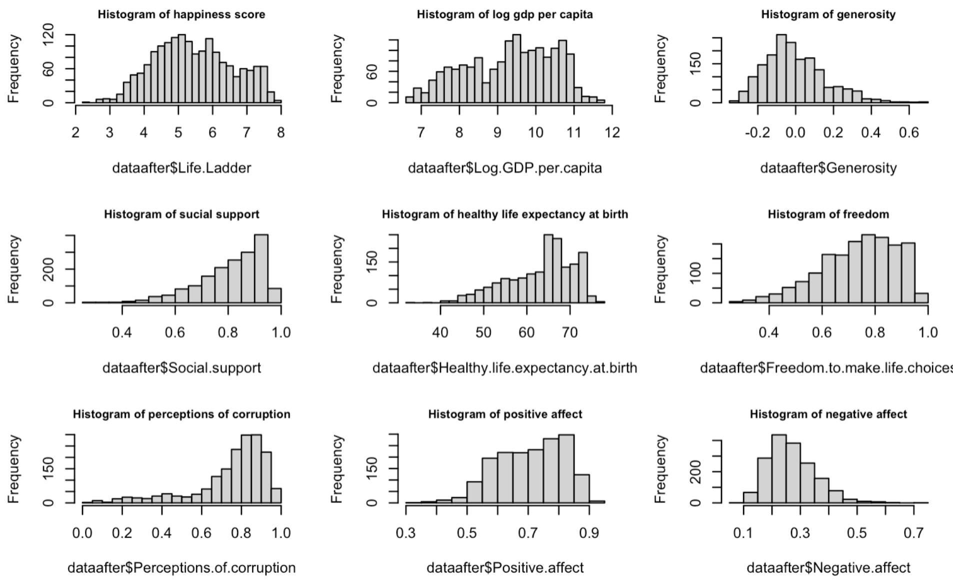

Figure 8: Distribution of All the Variables

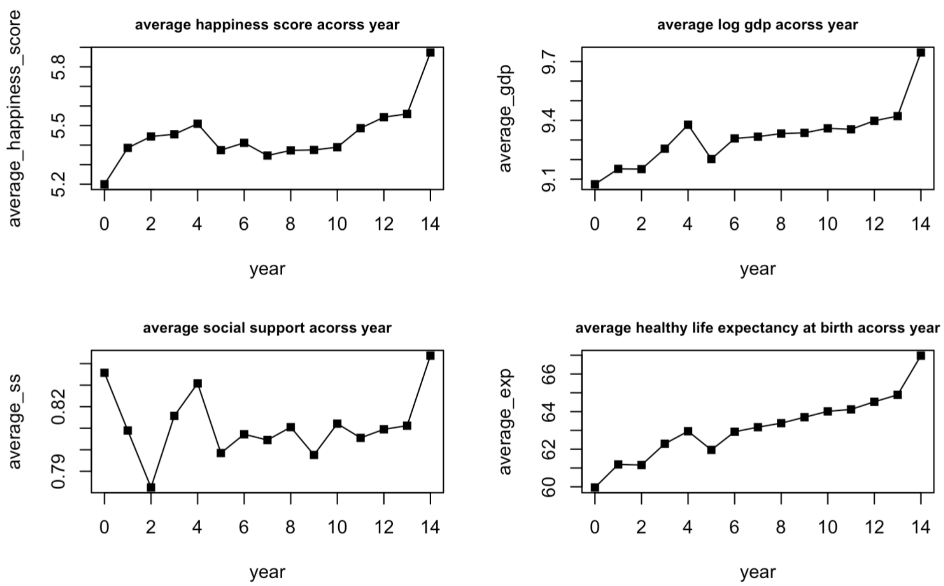

Figure 9: Time Trends of Variables Across All the Countries

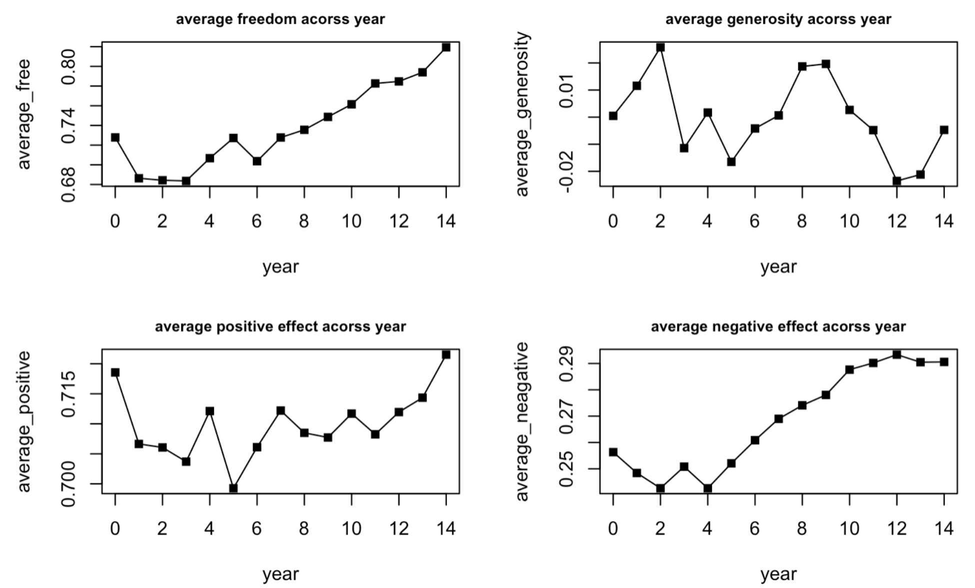

Figure 10: Time Trends of Variables Across All the Countries

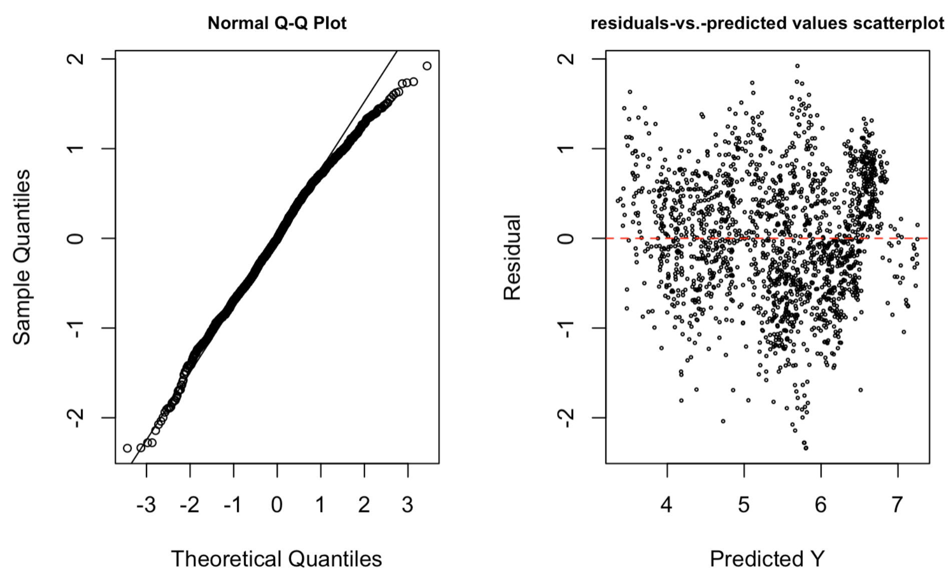

Figure 11: Assumptions check of model 1

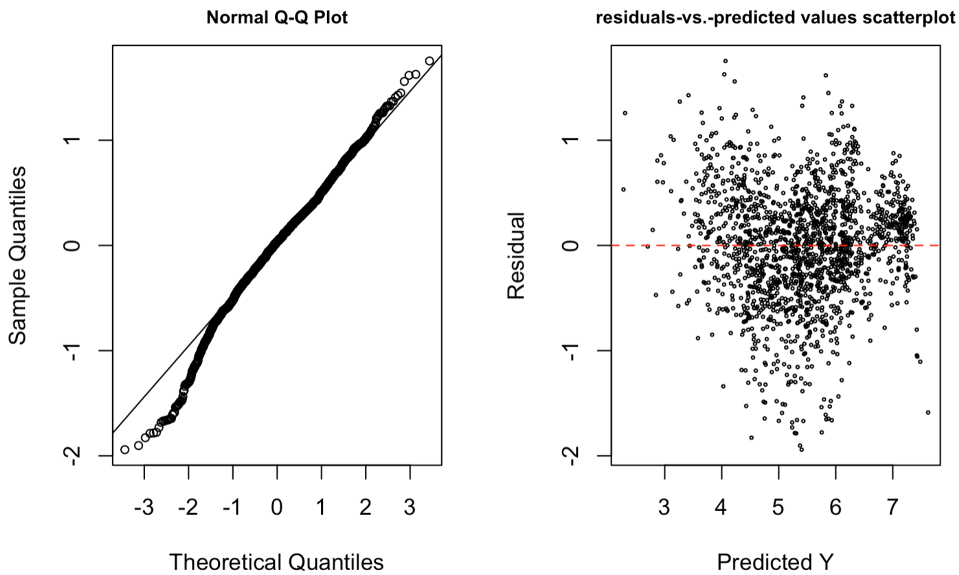

Figure 12: Assumptions check of model 2

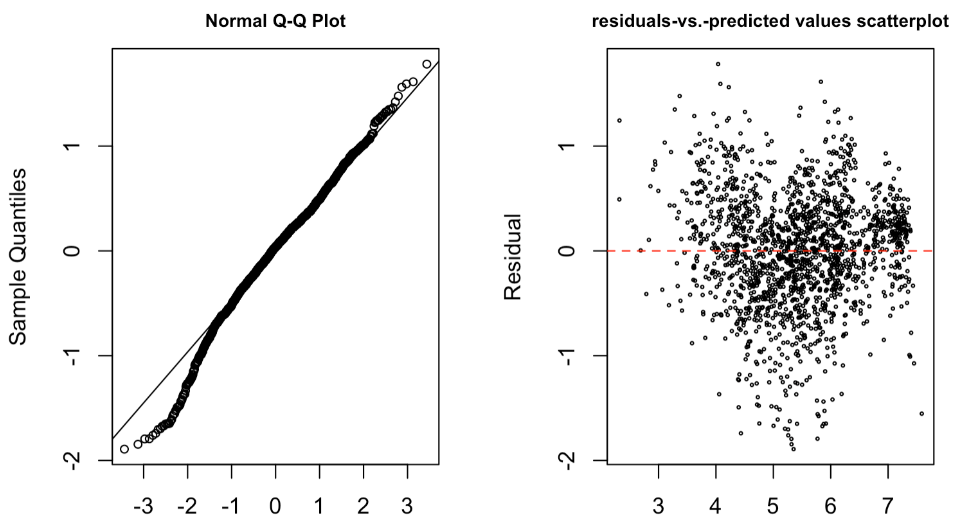

Figure 13: Assumptions check of model 3

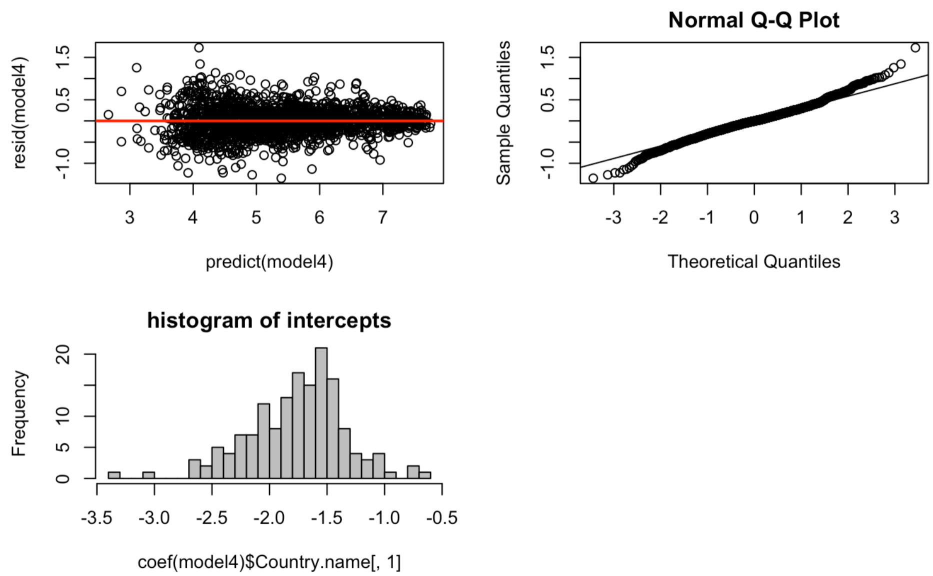

Figure 14: Assumptions check of model 4

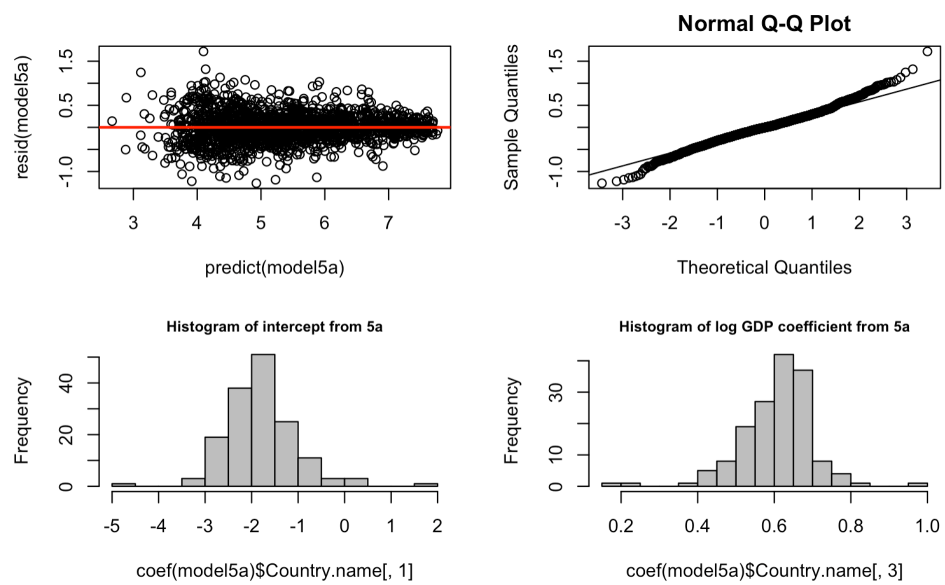

Figure 15: Assumptions check of model 5a

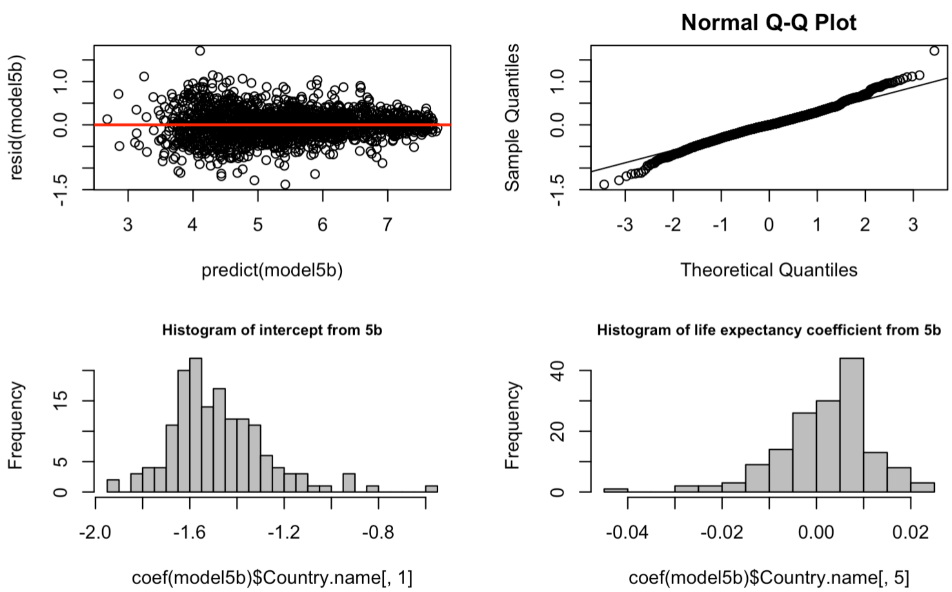

Figure 16: Assumptions check of model 5b

model 1				
Coefficients	Estimate	t value	p value	Level of significance
(Intercept)	-1.8007	-13.24	<2e-16	***
Log.GDP.per.capita	0.77741	53.69	<2e-16	***
model 2				
Coefficients	Estimate	t value	p value	Level of significance
(Intercept)	-2.525095	-13.55	< 2e-16	***
Log.GDP.per.capita	0.38226	14.997	< 2e-16	***
Social.support	1.843712	10.521	< 2e-16	***
Healthy.life.expectancy.at.birth	0.026743	7.711	2.11E-14	***
Freedom.to.make.life.choices	0.38979	2.98	0.00293	**
Generosity	0.418529	4.53	0.00000631	***
Perceptions.of.corruption	-0.69869	-7.939	3.67E-15	***
Positive.affect	1.987522	11.685	< 2e-16	***
Negative.affect	0.185287	0.993	0.32101	
model 3				
Coefficients	Estimate	t value	p value	Level of significance
(Intercept)	-2.563646	-13.77	< 2e-16	***
year	-0.011866	-3.307	0.000963	***
Log.GDP.per.capita	0.375162	14.709	< 2e-16	***
Social.support	1.826924	10.451	< 2e-16	***
Healthy.life.expectancy.at.birth	0.028465	8.14	7.58E-16	***
Freedom.to.make.life.choices	0.52099	3.821	0.000138	***
Generosity	0.386408	4.171	0.0000318	***
Perceptions.of.corruption	-0.704103	-8.023	1.91E-15	***
Positive.affect	1.93734	11.377	< 2e-16	***
Negative.affect	0.32881	1.721	0.085508	.
*notes:	0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1			

Figure 17: table1. summary of OLS model

model 4						
Random Effects				Fixed Effects		
Groups	Name	Variance	Std.Dev.	Coefficients	Estimate	t value
Country.name	(Intercept)	0.1984	0.4454	(Intercept)	-1.758293	-4.991
Residual		0.1253	0.354	year	0.001036	0.348
				Log.GDP.per.capita	0.585618	12.593
				Social.support	1.378043	7.259
				Healthy.life.expectancy.at.birth	-0.003627	-0.595
				Freedom.to.make.life.choices	0.511179	3.758
				Generosity	0.354784	3.07
				Perceptions.of.corruption	-0.602148	-4.517
				Positive.affect	1.607494	8.397
				Negative.affect	-0.844417	-4.346
model 5a						
Random Effects				Fixed Effects		
Groups	Name	Variance	Std.Dev.	Coefficients	Estimate	t value
Country.name	(Intercept) 2.93633	2.93633	1.7136	(Intercept)	-1.797161	-4.709
	Log.GDP.per.capita 0.03734	0.03734	0.1932	year	0.001442	0.482
Residual		0.12234	0.3498	Log.GDP.per.capita	0.606136	12.137
				Social.support	1.366867	7.188
				Healthy.life.expectancy.at.birth	-0.005783	-0.937
				Freedom.to.make.life.choices	0.509845	3.754
				Generosity	0.357655	3.073
				Perceptions.of.corruption	-0.614706	-4.428
				Positive.affect	1.578347	8.235

Figure 18: table2. summary of mixed effect model

Anova1							
Model	npar	AIC	BIC	Chisq	Df	Pr(>Chisq)	Level of significance
model4	12	1749.2	1814.5				
model5a	14	1749.2	1825.4	3.9424	2	0.1393	
Anova2							
model4	12	1749.2	1814.5				
model5b	14	1711.5	1787.7	41.709	2	8.77E-10	***
*notes:	0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1						

Figure 19: table3. summary of ANOVA